Electrical Circuits (2)

Lecture 2
Resonance

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Main Topics

1. Resonance
2. Magnetically Coupled Circuits
3. Three-Phase Circuits
4. Transient Analysis

References

A. Fundamentals of Electric Circuits (Alexander and Sadiku)
B. Principles of Electric Circuits (Floyd)
C. Circuit Analysis – Theories and Practice (Robinson & Miller)
D. Introductory Circuit Analysis (Boylestad)

21C

Resonance

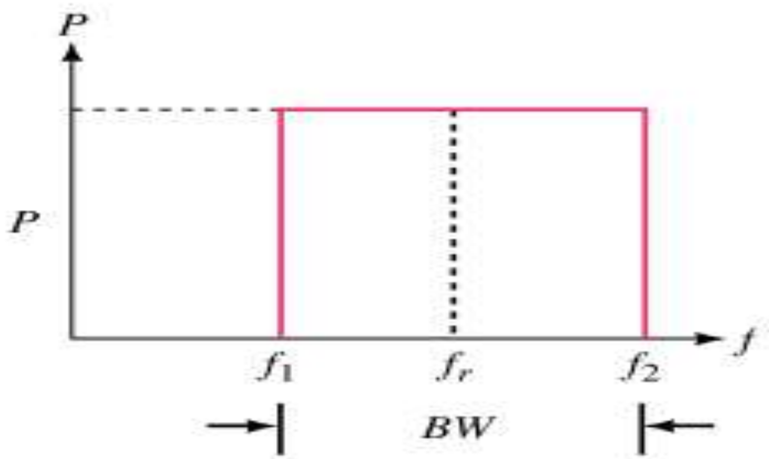
Circuits with both inductance and capacitance can exhibit a property called "Resonance" which is important in many applications

- Resonance is the basis for frequency selectivity in communication systems
- The ability of a radio or TV receiver to select a certain frequency (station) and at the same time eliminate frequencies from other stations is based on the principle of resonance

In this chapters
we will observe how resonant circuits are able to pass a desired range of frequencies from a signal source to a load.

Resonance

✓ In order to obtain all the transmitted energy for a given radio station or television channel, we would like a circuit to have the frequency response shown in Figure 21-1.a:



(a) Ideal frequency response curve

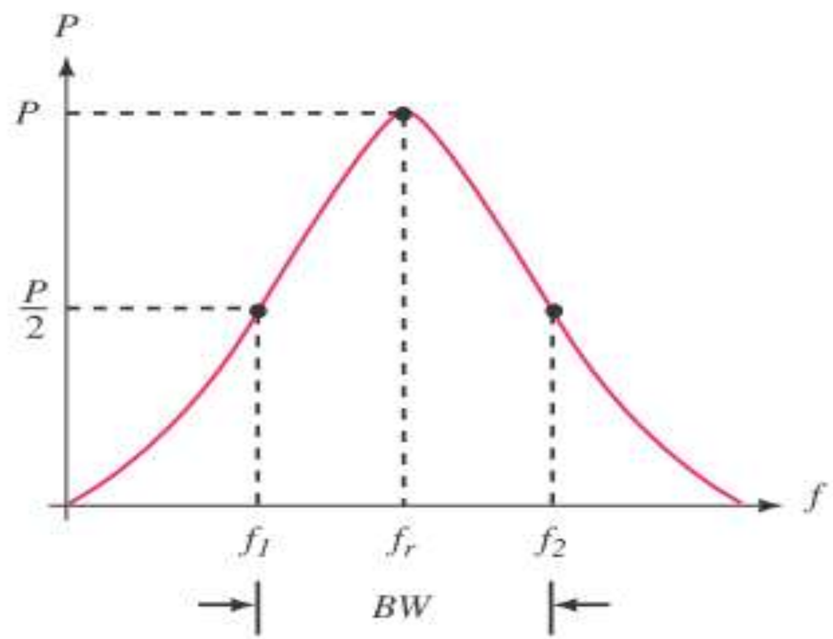
- fr : center frequency = station carrier frequency
- BW: bandwidth of the station = The difference between the upper and lower frequencies that we would like to pass

A circuit having an ideal frequency response would pass all frequency components in a band between f1 and f2 , while rejecting all other frequencies.

Resonance

Whereas there are various configurations of resonant circuits, they all have several common characteristics.

1. The resonant circuit consists of at least an **inductor** and a **capacitor** together with a **voltage or current source**.
2. Have a bell-shaped response curve centered at some resonant frequency as in shown in figure
3. This curve indicates that power will be a maximum at fr and varying the frequency in either direction results in a reduction of the power.

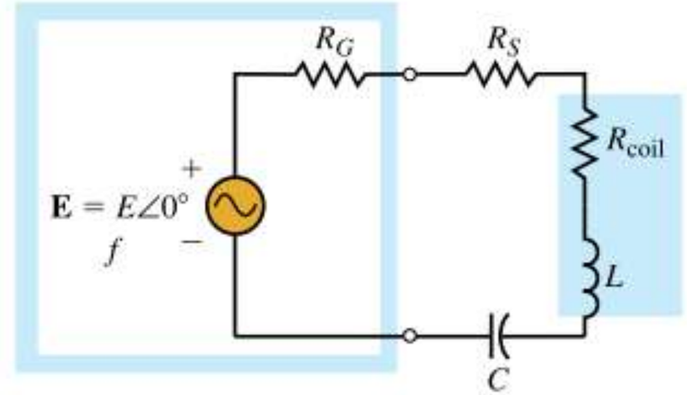


(b) Actual response curve of a resonant circuit

The bandwidth = the difference between the half-power points on the response curve of the filter.

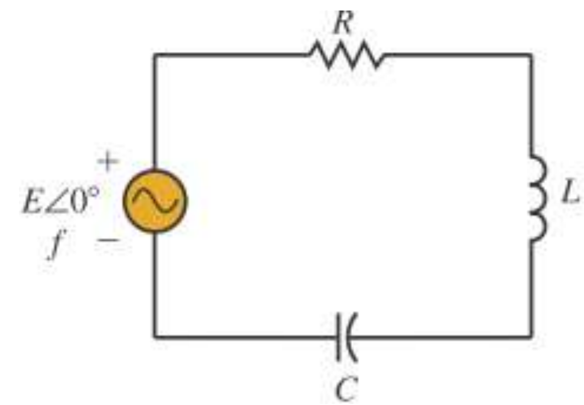
21.1 Series Resonance

- R_G : Generator resistance
- R_s : Series resistance
- Rcoil: Inductor coil resistance



In this circuit, the total resistance is expressed as

$$R = R_G + R_S + R_{coil}$$

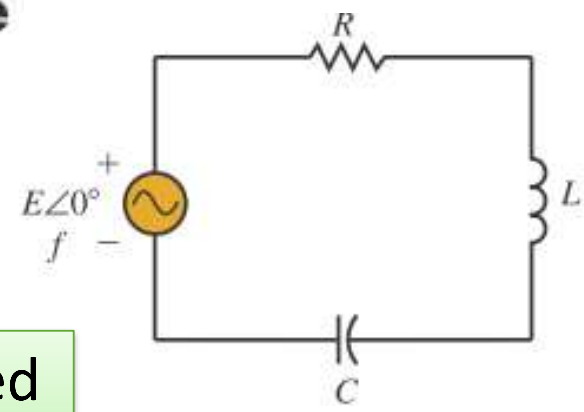


The total impedance is given by:

$$\mathbf{Z}_T = R + jX_L - jX_C$$
$$= R + j(X_L - X_C)$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Resonance is a condition in an *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.

21.1 Series Resonance



By setting the reactance of the capacitor and inductor equal to one another, the total impedances given by:

$$Z_T = R$$

The value of ω that satisfies this condition is called the resonant frequency

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega_S = \frac{1}{\sqrt{LC}} \quad \text{(rad/s)}$$

$$f_S = \frac{1}{2\pi\sqrt{LC}} \quad \text{(Hz)}$$

OR

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

At resonance the total impedances given by:

$$Z_T = R$$

At resonance, the total current in the circuit is determined from Ohm's law as

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{E\angle 0^\circ}{R\angle 0^\circ} = \frac{E}{R}\angle 0^\circ$$

The voltage across each of the elements in the circuit as follows:

$$\mathbf{V}_R = IR\angle 0^\circ$$

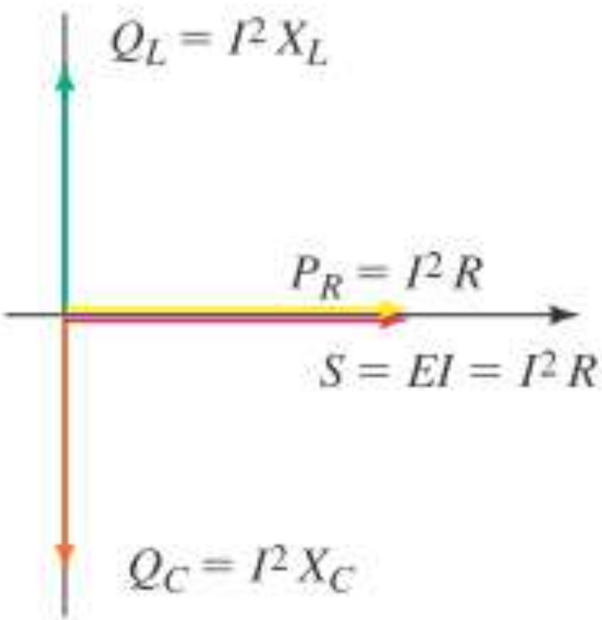
$$\mathbf{V}_L = IX_L\angle 90^\circ$$

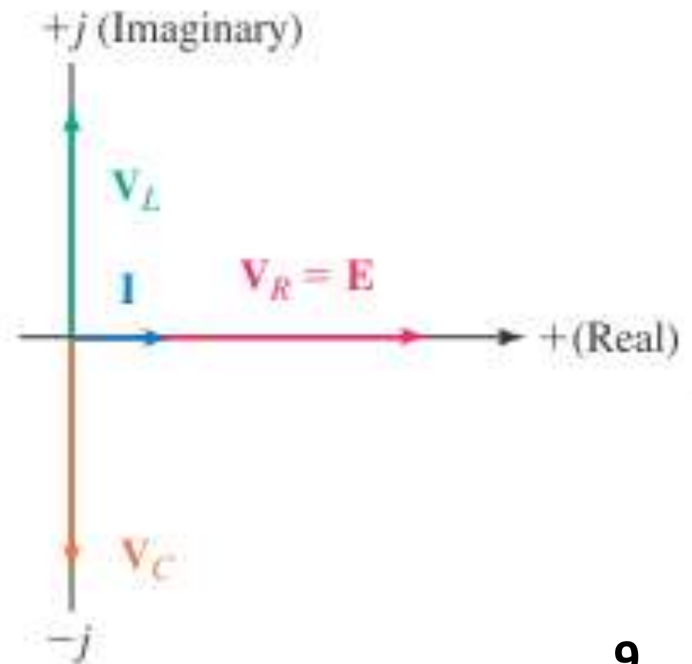
$$\mathbf{V}_C = IX_C\angle -90^\circ$$

$$P_R = I^2R \quad \text{(W)}$$

$$Q_L = I^2X_L \quad \text{(VAR)}$$

$$Q_C = I^2X_C \quad \text{(VAR)}$$



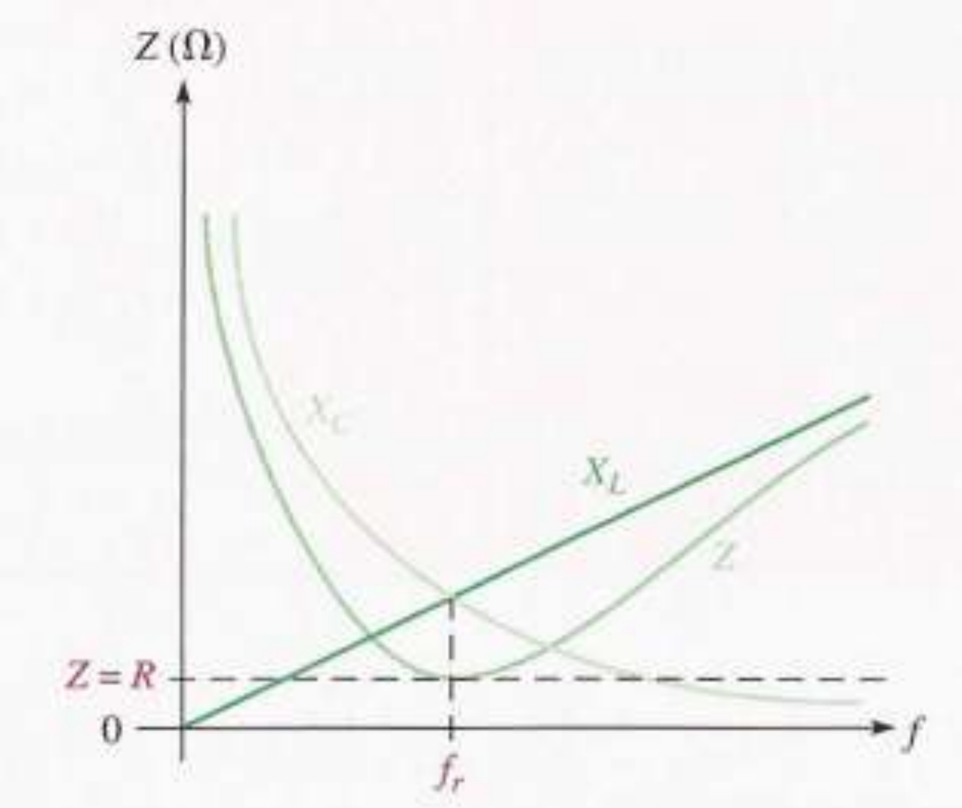


Impedance of a Series Resonant Circuit versus Frequency

Because the impedances of (L and C) are dependent upon frequency, the total impedance of a series resonant circuit must similarly vary with frequency

$$\mathbf{Z}_T = R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right)$$



Impedances Magnitude:

$$Z_T = \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2}$$

Impedances Angle:

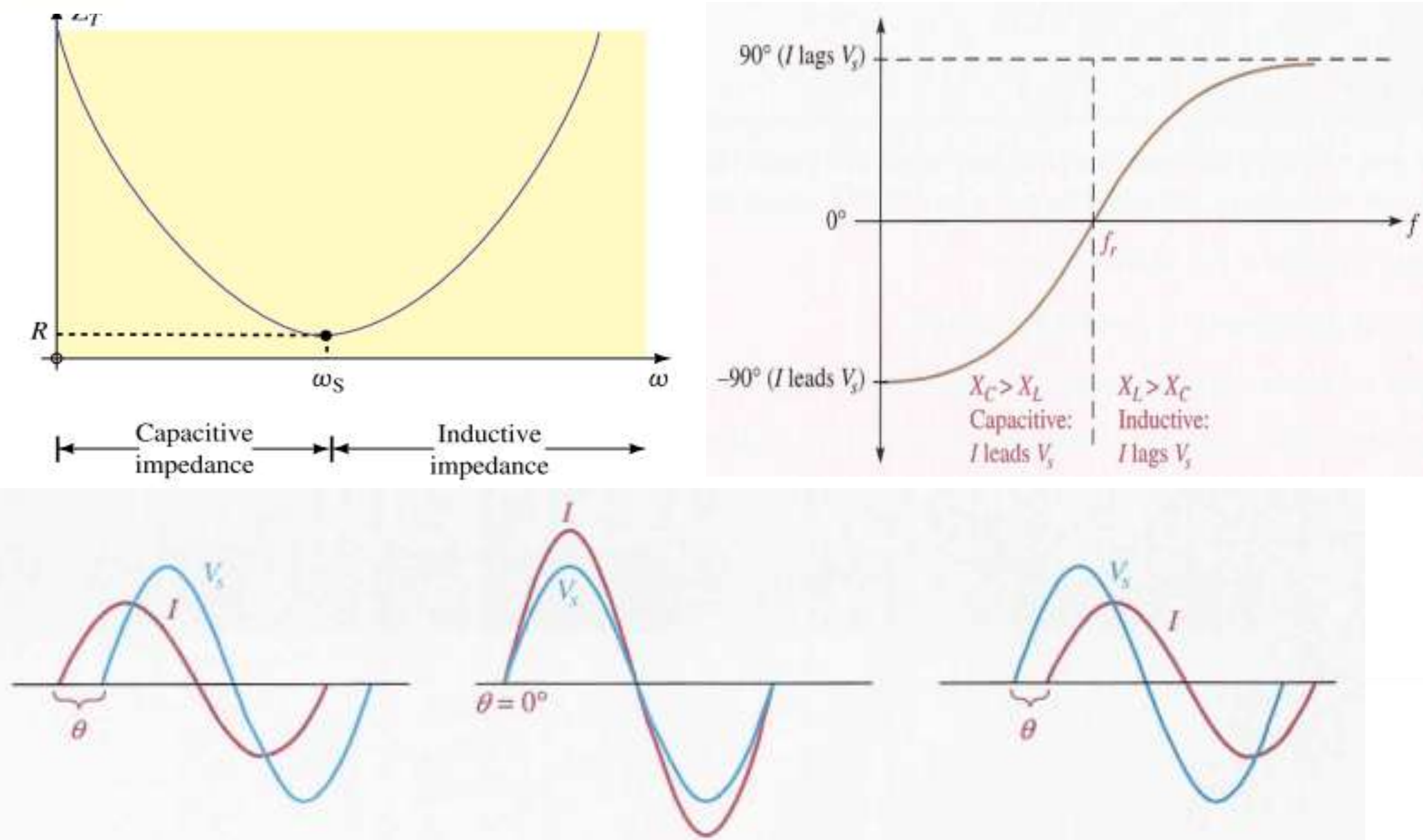
$$\theta = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$

When $\omega = \omega_S$:

$$Z_T = R \qquad \theta = \tan^{-1}0 = 0°$$

Impedance of a Series Resonant Circuit versus Frequency

FIGURE 21-7 Impedance (magnitude and phase angle) versus angular frequency for a series resonant circuit.



(a) Below f_r, I leads V_s.
(b) At f_r, I is in phase with V_s.
(c) Above f_r, I lags V_s.

Current and Power in a Series Resonant Circuit

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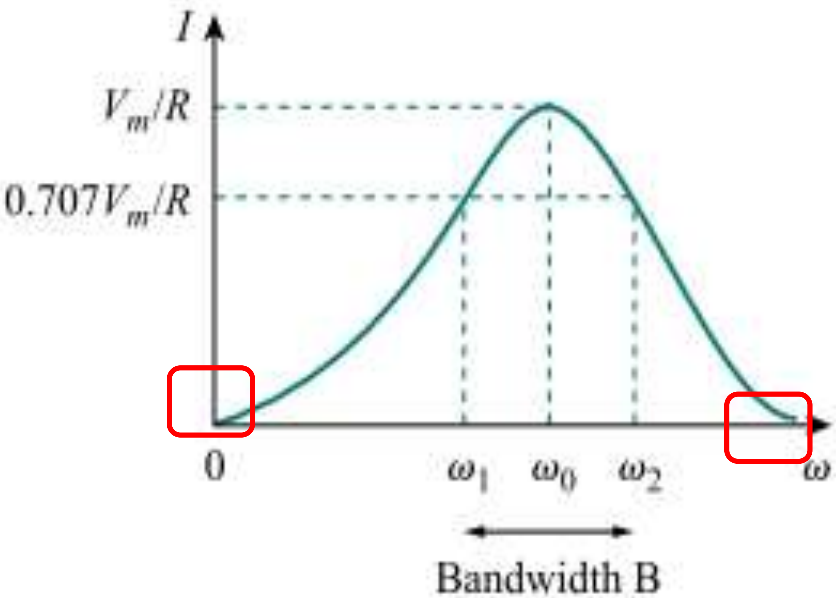
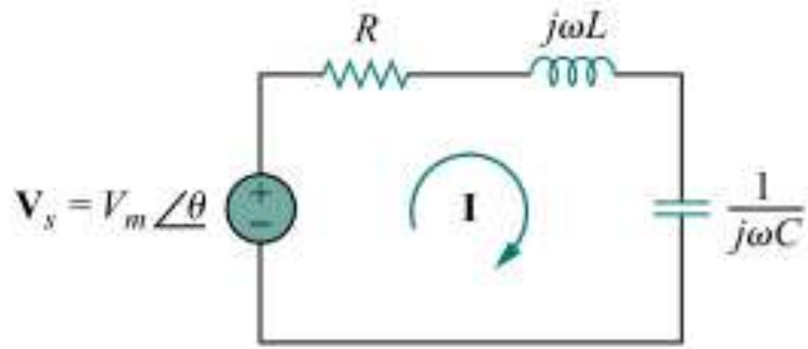
In this section, we examine how current and power are affected by changing the frequency of the voltage source.

Applying Ohm's law gives the magnitude of the current at resonance as

$$I_{max} = \frac{V_m}{R}$$





The frequency response of the circuit's current magnitude

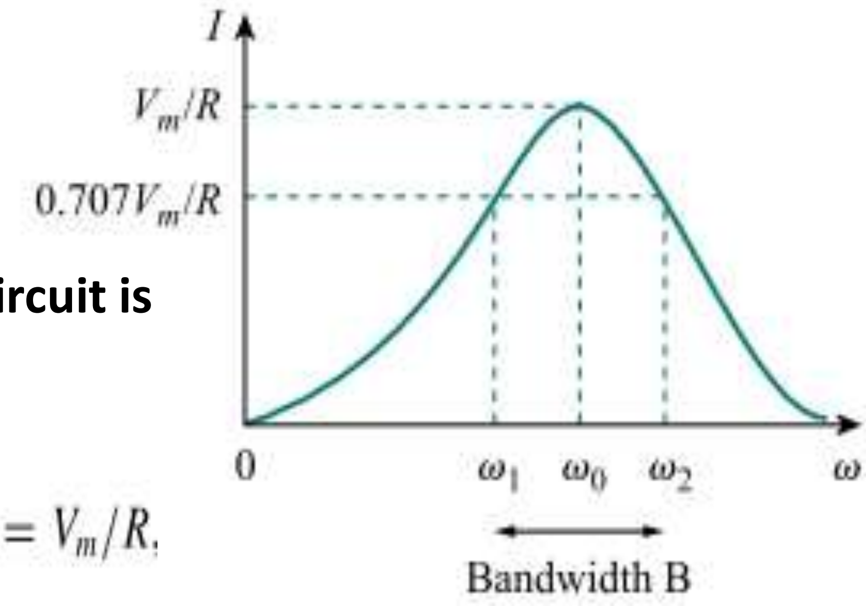
$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

For all other frequencies, the magnitude of the current will be less than Imax because the impedance is greater than at resonance.

Current and Power in a Series Resonant Circuit

P.602.C

Since the current is maximum at resonance, it follows that the power must similarly be **maximum** at resonance.



- **The average power dissipated by the RLC circuit is**

$$P(\omega) = \frac{1}{2} I^2 R$$

The highest power dissipated occurs at resonance, when $I = V_m/R$.

$$P(\omega_0) = \frac{1}{2}\frac{V_m^2}{R}$$

- **At certain frequencies** $\omega = \omega_1, \omega_2,$ **the dissipated power is half of that max**

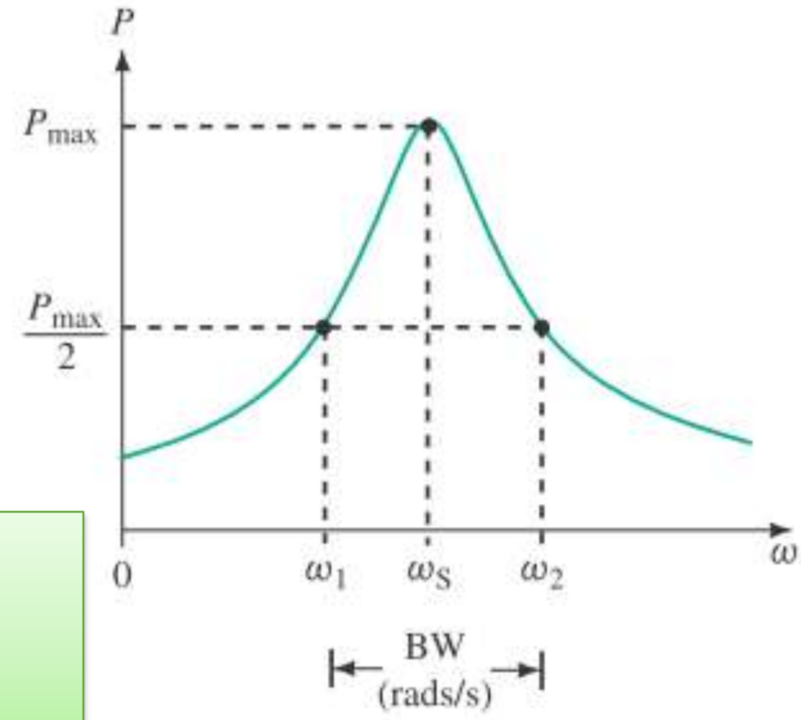
$$P(\omega_1) = P(\omega_2) = \frac{V_m^2}{4R}$$

They called the half-power frequencies (Points)

The Bandwidth – Selectivity – Quality Factor

Half-Power Frequencies (Points)
Cutoff Frequencies
Band frequencies

- ✓ This bell-shaped curve is called the selectivity curve



- ✓ Examining this curve, we see that only frequencies around Ws will permit significant amounts of power.

The Bandwidth of the resonant circuit (BW)

The difference between the frequencies at which the circuit delivers half of the maximum power.

$$BW = \omega_2 - \omega_1$$

It is called Half-Power Bandwidth

The Bandwidth – Selectivity – Quality Factor

- ✓ If the bandwidth of a circuit is kept very narrow, the circuit is said to have a **high selectivity,**

since it is highly selective to signals within a very narrow range of frequencies.

- ✓ On the other hand, if the bandwidth of a circuit is large, the circuit is said to have a **low selectivity**.

The elements of a series resonant circuit determine:

- The frequency at which the circuit is resonant
- The **shape** (and hence the **bandwidth**) of the power response curve.

1. If R and ws are kept constant:

- ✓ By **increasing the ratio of L/C,** the sides of the power response curve become steeper (i.e. decrease in the bandwidth)
- ✓ Inversely, decreasing the ratio of L /C causes the sides of the curve to become more gradual (i.e. increased bandwidth).

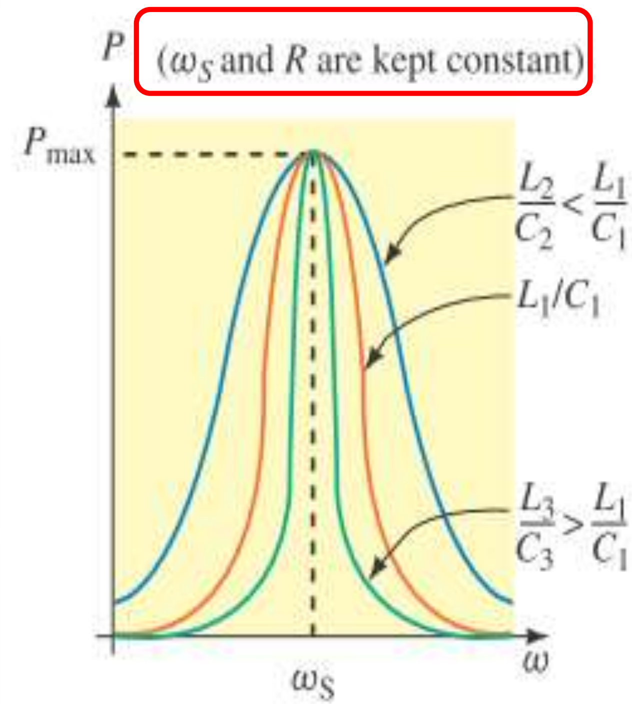
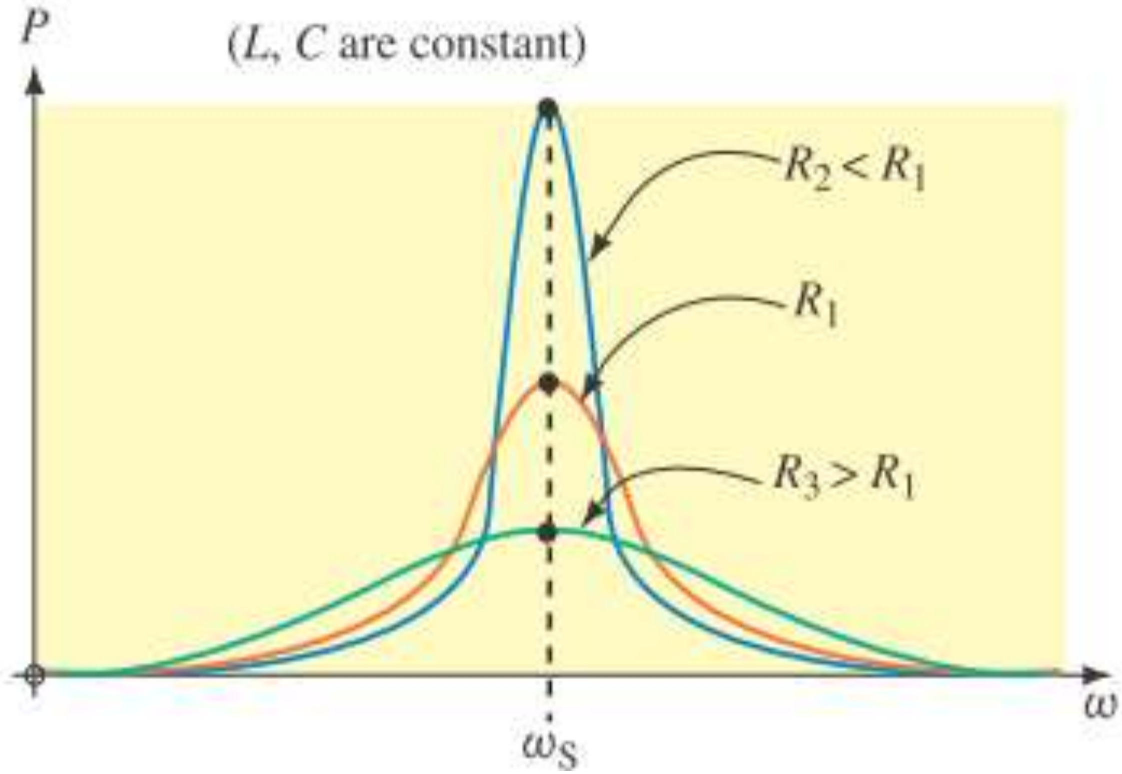


FIGURE 21–10

The Bandwidth – Selectivity – Quality Factor

2. If L and C are kept constant:



- ✓ The bandwidth is directly proportional to R
- ✓ The height of the curve is inversely proportional to R

A series circuit has the **highest selectivity** if the **resistance** of the circuit is kept to a **minimum**.

The Bandwidth – Selectivity – Quality Factor

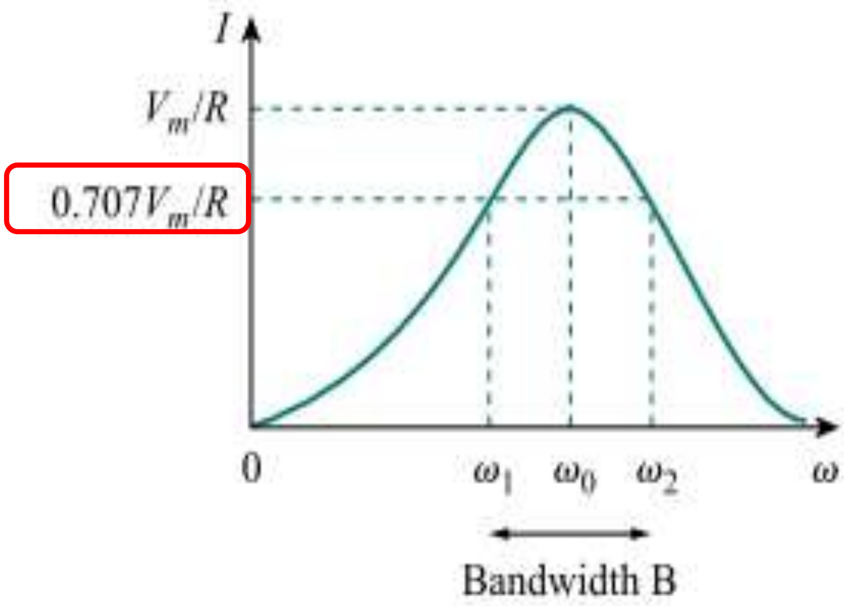
The half-power frequencies are obtained by setting Z equal to $\sqrt{2}R$,

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

Solving for ω, we obtain

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$



$$\text{BW} = \omega_2 - \omega_1$$

$$= \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} - \left(-\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\right)$$

$$\text{BW} = \frac{R}{L} \quad \text{(rad/s)}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

The resonant frequency is the geometric mean of the half-power frequencies.

The Bandwidth – Selectivity – Quality Factor

- The “sharpness” of the resonance in a resonant circuit is measured quantitatively by the **quality factor Q.**

Q: relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}$$

$$Q = \frac{\text{reactive power}}{\text{average power}}$$

Notice that the quality factor is dimensionless.

Q_L is equal to the Q_C at resonance,

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f)} = \frac{2\pi f L}{R}$$

$$Q_S = \frac{I^2X_L}{I^2R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

The Bandwidth – Selectivity – Quality Factor

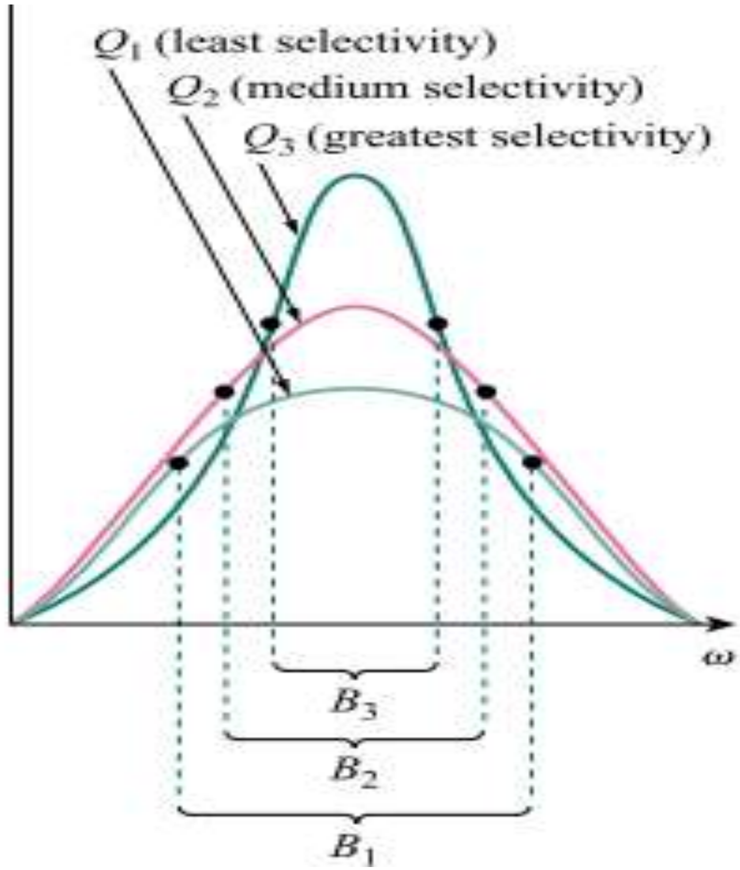
- **The relationship between the bandwidth B and the quality factor Q:**

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

So

$$B = \frac{R}{L} = \frac{\omega_0}{Q} = \omega_0^2 C R$$

$$Q = \frac{\omega_0}{B}$$



The quality factor of a resonant circuit is the ratio of its resonant frequency to its bandwidth.

- The higher the value of Q, the **more selective** the circuit is but the smaller the bandwidth.

The Bandwidth – Selectivity – Quality Factor

The selectivity of an RLC circuit
is the ability of the circuit to respond to a certain frequency
and discriminate against all other frequencies.

If the band of frequencies to be selected or rejected is narrow,
the quality factor of the resonant circuit must be high.

high-Q means equal to or greater than 10.

High-Q circuits are used often in communications networks.

For high-Q , the power frequencies are, for all practical purposes, symmetrical around the resonant frequency and can be approximated as:

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

Series Resonance Circuit (Cont.)

Quality Factor (Different Formulas)

$$Q_s = \frac{\omega_s L}{R}$$

$$Q_s = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R}\left(\frac{1}{2\pi\sqrt{LC}}\right)L$$

$$= \frac{L}{R}\left(\frac{1}{\sqrt{LC}}\right) = \left(\frac{\sqrt{L}}{\sqrt{L}}\right)\frac{L}{R\sqrt{LC}}$$

$$Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Series Resonance Circuit (Cont.)

EXAMPLE 20.5 A series *R-L-C* circuit is designed to resonant at $\omega_s = 10^5$ rad/s, have a bandwidth of $0.15\omega_s$, and draw 16 W from a 120-V source at resonance.

a. Determine the value of *R*.
b. Find the bandwidth in hertz.
c. Find the nameplate values of *L* and *C*.
d. Determine the Q_s of the circuit.

$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

a. $P = \frac{E^2}{R}$ and $R = \frac{E^2}{P} = \frac{(120\text{ V})^2}{16\text{ W}} = \mathbf{900\ \Omega}$

b. $BW = 0.15 f_s$ $\qquad f_s = \frac{\omega_s}{2\pi} = \frac{10^5\text{ rad/s}}{2\pi} = 15{,}915.49\text{ Hz}$

$$BW = 0.15 f_s = 0.15(15{,}915.49\text{ Hz}) = \mathbf{2387.32\ Hz}$$

c. $BW = \frac{R}{2\pi L}$ and $L = \frac{R}{2\pi BW} = \frac{900\ \Omega}{2\pi(2387.32\text{ Hz})} = \mathbf{60\ mH}$

$f_s = \frac{1}{2\pi\sqrt{LC}}$ and $C = \frac{1}{4\pi^2 f_s^2 L} = \frac{1}{4\pi^2(15{,}915.49\text{ Hz})^2(60 \times 10^{-3}\text{ H})}$
$= \mathbf{1.67\ nF}$

d. $Q_s = \frac{X_L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi(15{,}915.49\text{ Hz})(60\text{ mH})}{900\ \Omega} = \mathbf{6.67}$

Series Resonance Circuit (Cont.)

Note that at resonance:

1. The impedance is purely resistive, thus, $\mathbf{Z} = R$. In other words, the LC series combination acts like a short circuit, and the entire voltage is across R.
2. The voltage $\mathbf{V}_s$ and the current $\mathbf{I}$ are in phase, so that the power factor is unity.
3. The inductor voltage and capacitor voltage can be much more than the source voltage.

- Point (3) can be verified by applying the voltage divider rule to the circuit of Fig. 20.2, we obtain

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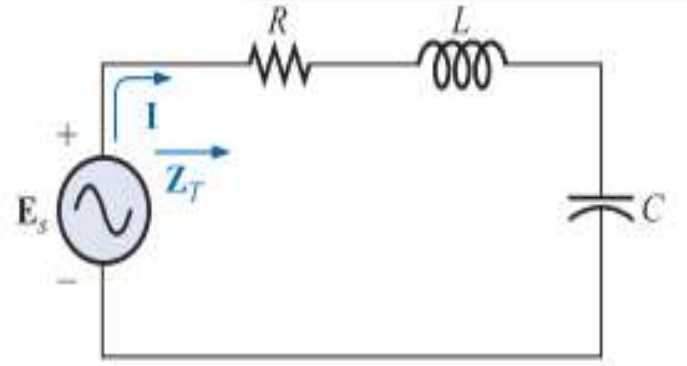
$$V_L = \frac{X_L E}{Z_T} = \frac{X_L E}{R} \quad \text{(at resonance)}$$

$$V_{L_s} = Q_s E$$

$$V_C = \frac{X_C E}{Z_T} = \frac{X_C E}{R}$$

$$V_{C_s} = Q_s E$$



Series Resonance Circuit (Cont.)

$$V_{L_s} = Q_s E \qquad V_{C_s} = Q_s E$$

- Since Q s is usually greater than 1, the voltage across the capacitor or inductor of a series resonant circuit can be significantly greater than the input voltage.